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An experimental study of impulsively started turbulent axisymmetric jets

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Abstract. An impulsively started turbulent jet injected into quiescent surroundings with a constant inlet velocity has been studied experimentally. Results show that the jet length increases linearly with the square-root of time, over a wide range of Reynolds number calculated with respect to the jet diameter. The celerity factor, x_f/tU, has been found to be nearly constant at 2.47 throughout with a 5% variance. Here, x_f is the jet length, t is the time and U is the jet exit velocity. These results compare favourably with earlier results reported at lower Reynolds numbers. Finally, we present a simple model based on the integral energy balance of the turbulent boundary layer equation for an impulsively started turbulent axisymmetric jet. The model predicts a jet length that scales as, $(x_f/d) = \sqrt{(9B/10)(tU/d)}$ where d is the nozzle diameter and $B(\approx 6.0)$ is the velocity-decay constant. This gives a celerity factor, $\alpha \equiv \sqrt{9B/10} = 2.32$ in close agreement with the experiments.

PACS. 47.27.wg Turbulent jets – 47.27.nb Boundary layer turbulence

An important property of turbulence lies in its ability to transport and mix fluid much more effectively than a comparable laminar flow. The shearing encountered by the jet-fluid as it traverses across the surrounding fluid gives the jet its characteristic mixing property. Industrially, it is this property that has been used extensively to achieve rapid fluid-fluid mixing. For example, the efficacy of any jet-engine, which is so heavily dependent on proper fuel-air mixing, has been found to be vastly improved by the selection of a proper fuel-jet orientation.

The turbulence characteristics of steady axi-symmetric turbulent jets have been well documented through a number of theoretical and experimental investigations. Structurally, a jet comprises of a potential-flow core that slowly shrinks from the near-exit region to the farther regions. Simultaneously, the peripheral regions become unstable leading to eddy-formation as the jet travels downstream. For sufficiently high Reynolds numbers ($Re \equiv Ud/\nu \gtrsim 10^3$), the whole body of the jet turns turbulent at a distance of about 4 diameters from the jet exit. Here, U is the average jet velocity at the nozzle exit of diameter d and ν is the fluid kinematic viscosity. Further, the experiments [1–4] show that the mean axial and radial velocity profiles are self similar for distances beyond 30 diameters from the nozzle exit and are independent of Re.

Most experimental studies involving unsteady turbulent jets have been mainly qualitative in nature with only a few measuring the evolving jet velocity profiles in the far field region. A recent experimental study by Ai et al. [5] measured the jet penetration length for a suddenly started axisymmetric turbulent water jet and compared the measurements with those reported previously to find that while the dimensionless penetration length (rendered dimensionless with d) scaled as the square root of dimensionless time (rendered dimensionless with d/U) in all cases, the proportionality coefficient varied due to differences in experimental conditions among various experimental studies. The most extensive among those reported are by Johari et al. [6] who studied the mixing characteristics of an impulsively started water jets over a wide range of jet Reynolds number and nozzle diameters and found the coefficient to be a constant. The location of the jet-tip was found to vary linearly with the square root of the normalized time, with the proportionality constant being termed as the *celerity* parameter $\alpha \equiv x_f/tU$, where x_f is the jet length. It was found that the value of α for a fluid element in the steady region of the evolving jet was approximately half the value of that measured at the jet tip. While α showed some variations with changing nozzle time (defined as $\tau \equiv d/U$), the variations were within the range of experimental errors, and no general dependence could be ascertained. Results were reported for Reynolds numbers up to 20 000 and α varied by 10% about the average value of 2.4.

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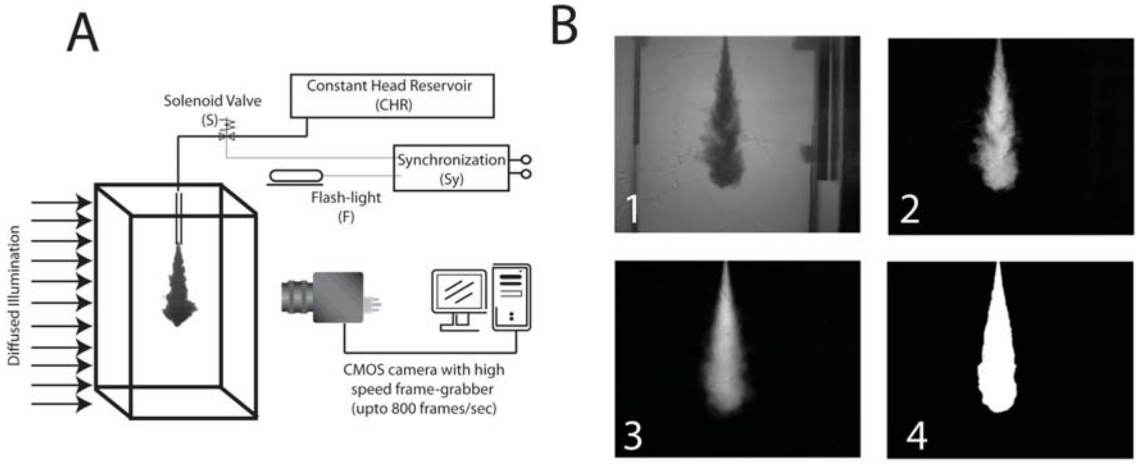

Fig. 1. Schematic for experimental set-up and sequence of analysis. (A) Images were captured on a CMOS camera at 800 fps; vertical resolution ranged between 1.5 μm/pixel and 3 μm/pixel. Diffused light was used to uniformly illuminate the flow-area (between 30 d and 60 d downstream). A flash-induce synchronization (estimated ± 4 ms jitter) was used to actuate an electrically operated solenoid valve, thereby introducing the dyed-jet impulsively into the quiescent mass of water contained in a tank. A constant head reservoir(CHR) ensured $<0.001\%$ variation in flow. (B) shows the various stages of image analysis. The raw image (B.1) was corrected for background (B.2), averaged over several identical runs (B.3) and thresholded (B.4) to identify the jet-tip.

On the theoretical front, the only existing model is that of Witze [7] who formulated the turbulent starting jet in terms of a spherical vortex attached to the jet and performed a mass and momentum balance on the vortex sphere to determine the speed of the jet tip. Though the model was similar to that of Turner [8] for starting plumes and Abramovich and Salon [9] for starting laminar jets, an empirical relation for the drag on the spherical vortex was used to correlate the data[1]. The predicted jet penetration length followed the expected square root dependence on time but the coefficient was dependent on the ratio of nozzle diameter and average jet exit velocity. Recall that Johari et al. [6] found the coefficient to be independent of Reynolds number and nozzle diameter.

Given the discrepancy between the predictions of Witze's model and the existing experimental results, we were motivated to examine the flow-field evolution for an impulsively started axisymmetric turbulent jet, introduced into quiescent surroundings, over a larger range of Reynolds numbers than reported previously. A novel approach in synchronization of the imaging with the start of the jet flow reduced the uncertainty in the measurements compared to earlier studies. Our experiments show that the celerity parameter is indeed constant for a given jet and ambient fluid and does not depend on the nozzle time for the given range of Reynolds numbers, as suggested by the model proposed earlier by Witze [7].

The discrepancy between Witze's model predictions and results from our experiments suggest a weakness in Witze's model, perhaps arising from the empirical relation introduced to acccount for the drag over a spherical vortex. This discrepancy motivated us to model the starting-jet problem from the perspective of self-similarity. The modeling approach followed here is different in that we assume time dependent velocity profiles that resemble the self similar form and use integral energy balance to predict the evolving jet front. The model predicts a discontinuous jet front that separates the quiescent region in the ambient fluid from that in the jet. An analytical expression for the jet penetration length was derived that not only predicts the expected scaling but also agrees closely with the experiments. The ratio of penetration length to square root of time is a constant and is related to the velocity decay constant of a steady turbulent jet. Notably, no dependence of the celerity-parameter upon the nozzle time is suggested, as opposed to predictions of Witze's model and in agreement with our experimental observation.

1 Experimental set-up

The evolution of an impulsively started turbulent water-jet introduced into a quiescent bulk of water was studied using visualization techniques. Figure 1 shows a schematic of the hydraulic assembly. A constant head reservoir was installed at fixed heights so as to ensure a constant flow at the nozzle-outlet. The nozzle diameter in all experiments was 5 mm. The nozzle was designed so as to eliminate any swirling motion that might get induced in the jet-fluid while emerging from the nozzle. The Reynolds numbers were kept high so as to ensure that the jet emerging out of the nozzles is fully turbulent. A large neutral-glass flow-tank of dimensions $0.5\ \text{m} \times 0.5\ \text{m} \times 1.0\ \text{m}$ was chosen so that the side walls are far from the nozzle and have negligible influence on the flow. The flow was visualized by enhanced contrast-imaging using a colored dye

[1] The more recent theoretical models on the start-up of jets also focus in the region close to the nozzle exit where the flow is laminar [10,11].

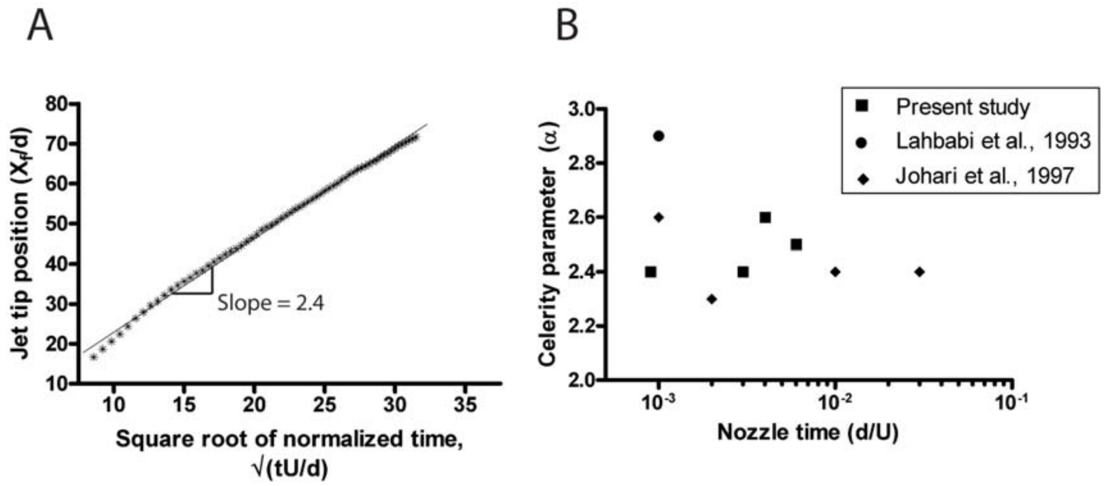

Fig. 2. (A) Position of the jet tip (marked by $*$), normalized by the nozzle diameter and plotted against square root of normalized time duration of flow, for a Reynolds number of 8866. The celerity parameter α was obtained as the slope of the linear fit (—) and was found to be 2.4 for the flow corresponding the above plot. (B) shows the variation of celerity parameter with nozzle time as measured in present study as well as those reported earlier. Notably, no trend for the variation of celerity parameter with the nozzle time can be discerned.

(Camel™ Black Ink, with Schmidt number of approximately 30 [12]) as the jet fluid and imaging the flow against a bright background.

The dye-containing jet-fluid was impulsively injected into the quiescent waters of the glass-tank using the solenoid valve with an opening time of a few milliseconds. It was ensured that for each run, the exact time instance when the solenoid valve was opened was recorded with an accuracy of 1 ms, by firing a flash-light (Vivitar 6F) in parallel with the opening of the valve. Since both the solenoid valve and the flash were switched on at the same instance and the camera was kept on throughout this process, we could capture images at exactly the same instances, for each of the runs, at any given Reynolds number. With the opening time of the solenoid-valve being rated at 3 ms and the response time of the flash being rated at 1 ms, the uncertainty of each run was limited to about 4 ms. This is in contrast to the experiments of Johari et al. [6] wherein an uncertainty of about 33 ms has been reported. The use of a flash to record the exact moment at which the valve is actuated, circumvents this problem.

Seven such visualizations were made for each Reynolds number under exactly similar flow conditions. All such visualizations were ensemble averaged and the averaged images thus obtained were analyzed to study the evolution of flow with time. The objective was to follow the evolution of the jet-tip and the radial spread with time. Table 1 details the geometry and flow parameters for the runs conducted.

Table 1. Flow parameters and imaging region for a nozzle of diameter, $d = 5$ mm.

S.No.	*Re.*	Runs	U (m/s)	Region imaged
1	4178	7	0.835	$9d$–$42d$
2	6576	7	1.315	$25d$–$75d$
3	8866	7	1.773	$15d$–$70d$
4	27 356	7	5.471	$15d$–$75d$

2 Experimental results

The position of the jet-tip in all our experiments increased linearly with the square-root of normalized time for $x_f/d \gtrsim 30$. This linearity was exhibited in the far-field at all the Reynolds numbers studied. Figure 2 shows the variation of the celerity-factor, α, with change in nozzle-time, τ. Although there was some variation of the celerity factor α with Re, the variation was well within experimental errors and no trend could be discerned. Our results exhibit a variance of 5% about the mean value of 2.47, over the set of Reynolds numbers ranging from 4000 to 27 000. This is roughly in agreement with the results of Johari et al. [6] wherein a relatively smaller range of Reynolds numbers was explored (5000–20 000) with a variance of 10% about the mean value of 2.4. Further, our results exhibit a more stable behaviour in terms of the position of the virtual origin, x_o (see next section). While in the experiments of Johari et al., the virtual origin fluctuated from being positioned in front of the nozzle to being behind it, our experiments consistently exhibit a negative intercept, signifying a virtual origin placed behind the nozzle-tip. The somewhat larger spread in the measured celerity factor by Johari et al. [6] may be attributed to the fact that the opening of the valve and the image-acquisition were not synchronized whereas the usage of a flash-signal in our case ensured proper synchronization between the two events.

Given the near constancy of the celerity factor (especially for higher Reynolds numbers), it may be concluded that the effect of inlet conditions (in the form of inlet velocity and nozzle diameter) on the advancement of the tip becomes negligible in the far-fields for such higher Reynolds numbers.

3 Model for an impulsively started turbulent jet

In this section we propose a new model to predict the jet length as a function of time for an impulsively started jet. As mentioned earlier, this is motivated by the discrepancy between Witze's predictions and our experimental results. Specifically, Witze [7] predicts the jet penetration length to vary with the square root dependence on time but the coefficient (celerity parameter) depended on the ratio of nozzle diameter and average jet exit velocity. On the contrary, our experiments show that the celerity factor is constant for our entire range of Reynolds numbers.

We start by noting that the dominant mean flow in a turbulent axisymmetric jet is in the direction of axis of the jet with the mean lateral velocity being small compared to the axial velocity and the axial gradients being negligible compared to the lateral gradients. As a result, the boundary layer approximation [13] can be applied to obtain the ensemble averaged continuity and axial momentum balance equation for the turbulent flow,

$$\frac{\partial \langle u \rangle}{\partial x} + \frac{1}{r}\frac{\partial (r\langle v \rangle)}{\partial r} = 0 \qquad (1)$$

$$\frac{\partial \langle u \rangle}{\partial t} + \langle u \rangle \frac{\partial \langle u \rangle}{\partial x} + \langle v \rangle \frac{\partial \langle u \rangle}{\partial r} = \frac{\nu_t}{r}\frac{\partial}{\partial r}\left(r \frac{\partial \langle u \rangle}{\partial r}\right) \qquad (2)$$

with the turbulent viscosity (ν_t) assumed constant throughout. Here, r is the radial coordinate, x is the axial coordinate, $u(x, r, t)$ and $v(x, r, t)$ are respectively the axial and radial velocities, and the angled brackets denote ensemble averaged quantities.

Multiplying (2) with $\langle u \rangle r$ and integrating the equation across a section of the jet gives the integral energy balance equation for an unsteady jet,

$$\frac{\partial}{\partial t}\left\{ \int_0^\infty \frac{r\langle u \rangle^2}{2} dr \right\} + \frac{\partial}{\partial x}\left\{ \int_0^\infty \frac{r\langle u \rangle^3}{2} dr \right\} = - \int_0^\infty \nu_t r \left(\frac{\partial \langle u \rangle}{\partial r}\right)^2 dr. \qquad (3)$$

We next assume that the velocity profiles throughout the jet follow the same self similar form as that found under steady state[13],

$$\langle u(x, r, t) \rangle = \frac{\langle u_c(x, t) \rangle}{(1 + a\eta^2)^2} \qquad (4)$$

which on substitution in (3) gives

$$\frac{\partial \langle u_c \rangle}{\partial t} + \frac{9}{10}\langle u_c \rangle \frac{\partial \langle u_c \rangle}{\partial x} = -\frac{3}{5}\left\{ \frac{\langle u_c \rangle^2}{x} + \frac{4a\nu_t \langle u_c \rangle}{x^2}\right\}. \qquad (5)$$

Here, $\eta \equiv r/(x - x_0)$ is the similarity variable, x_0 is the virtual origin, and u_c is the centerline axial velocity. Also, a is a constant, independent of jet Reynolds number and is directly related to the speading rate of the jet. As expected, (5) yields $u_c(x, t \to \infty) = 8a\nu_t/(x - x_0)$, the standard result for steady turbulent jets [13]. On scaling the length with the nozzle diameter, $\bar{x} \equiv x/d$, centerline velocity with nozzle exit velocity, $\bar{u} \equiv \langle u_c \rangle / U$ and time with d/U, we can write (5) in its non-dimensional form,

$$\frac{\partial \bar{u}}{\partial \bar{t}} + \frac{9}{10}\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} = -\frac{3}{5}\left\{ \frac{\bar{u}^2}{\bar{x}} + \frac{B}{2}\frac{\bar{u}}{\bar{x}^2}\right\}. \qquad (6)$$

Equation (6) predicts the evolving dimensionless centerline velocity as function of space and time. Interestingly, the equation suggests a dimensionless front velocity that is dependent only on the velocity decay constant, $B(\equiv 8a\nu_t/Ud)$. Since (6) admits a discontinuous solution with the discontinuity separating the steady jet from the ambient flow (zero velocity), the speed of the discontinuity (or jet tip speed) is given by the jump condition [14],

$$\frac{\mathrm{d}\bar{x}_f}{\mathrm{d}\bar{t}} = \frac{\mathcal{F}(\bar{u}|_{\bar{x}_f -}) - \mathcal{F}(\bar{u}|_{\bar{x}_f +})}{\bar{u}|_{\bar{x}_f -} - \bar{u}|_{\bar{x}_f +}} \qquad (7)$$

where $\mathcal{F}(\bar{u}) \equiv (9/20)\bar{u}^2$ and $\bar{x}_f$ is the position of the discontinuity with subscripts '−' and '+' denoting, respectively, the states on the left and right side of the discontinuity. On substituting the relevant quantities in (7), the jet tip velocity is obtained as,

$$\frac{\mathrm{d}\bar{x}_f}{\mathrm{d}\bar{t}} = \frac{9}{20}\frac{B}{\bar{x}_f} \qquad (8)$$

which on integration gives the dimensionless jet length,

$$\bar{x}_f = \left(\frac{9}{10} B \right)^{\frac{1}{2}} (\bar{t})^{\frac{1}{2}}. \qquad (9)$$

Thus the dimensionless jet penetration length scales with the square root of time (dimensionless) with the celerity parameter, $\alpha \equiv \sqrt{9B/10}$, related to the properties of the steady axisymmetric turbulent jet.

Measurements [2–4] for steady turbulent jets with Reynolds number differing by a factor of almost ten ($Re = 1 \times 10^4$–9.5×10^4) show that small differences in the measured values of B(=5.9–6.06) are within experimental uncertainties. Consequently, for $B \approx 6.0$, the jet penetration length (dimensionless) scales with the square root of dimensionless time with α =2.3. Our experiments show that $\alpha = 2.47$ with about a 5% scatter. Johari et al. [6] found that α varied from 2.14 to 2.58 over five experiments at varying Reynolds numbers ($Re = 0.5 \times 10^4$–2.0×10^4), nozzle velocities and nozzle diameters. Since the variation in the value of the proportionality constant was within experimental uncertainties, they assumed a constant value of 2.4 for all conditions. These measurements are remarkably close to the predicted value. Also, these results are in variance with the Witze's model [7] predictions that require the proportionality constant to vary with nozzle time, d/U. We believe that the latter is an artifact of the empirical relation introduced to account for the drag over the spherical vortex.

The success of our model stems from the self similarity of the mean axial and radial velocity profiles for fully turbulent jets and their independence of the jet Reynolds number. Further, the velocity decay constant (B) and the spreading rate are also independent of the jet Reynolds number. These properties are unique to the turbulent jets.

4 Conclusion

The development of an impulsively started turbulent liquid axisymmetric jet injected into a quiescent mass of liquid was studied experimentally. Flow visualization involving contrast imaging was used to study the flow for a wide range of Reynolds numbers. Results showed that while the jet tip advances linearly with square-root of time, the celerity parameter remains nearly independent of the nozzle-time. The advancement of the jet-tip was reported to exhibit a constant celerity factor α of 2.47 over a wider range of Reynolds numbers than that reported by Johari et al.[6]. Improved synchronization ensured that the scatter was only 5% of the mean value. Also, the virtual origin was consistently found to be located behind the nozzle for all Reynolds numbers.

Finally, we present a simple model based on the integral energy balance of the turbulent boundary layer equation that predicts the jet tip penetration velocity for an impulsively started turbulent jet. The analytical expression for the penetration length not only predicted the expected scaling but also agrees well with measurements.

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